Meeting Specifications is Not Good Enough—The Taguchi Loss Function

John McConnell, Brian K. Nunnally, and Bernard McGarvey

"Analysis and Control of Variation" is dedicated to revealing weaknesses in existing approaches to understanding, reducing, and controlling variation and to recommend alternatives that are not only based on sound science, but also demonstrably work. The objective of the column is to combine sound science with proven practical advice.

Reader comments, questions, and suggestions will help us fulfill our objective for this column. Please send your comments and suggestions to column coordinator John McConnell at john@wysowl.com.au or managing editor Susan Haigney at shaigney@advanstar.com.

KEY POINTS DISCUSSED

The following key points are discussed:

- Good science requires stable process operations with low variation
- It is good management to reduce the variation in any quality characteristic, whether this characteristic be in a state of control or not, and even if few or no defectives are being produced
- Taguchi's methods, Deming's approach, Six Sigma, and Little's Law all demonstrate that reducing variation is an economic imperative
- Many scientists and managers find themselves trapped in the old "goal post" mentality for specifications
- The Taguchi Loss Function approach demonstrates how reducing variation reduces costs, even if all outcomes meet specification
- Taguchi's concepts led to the development of the most complete definition of quality: All characteristics should have minimum, stable variation around an optimum value.

INTRODUCTION

Dr. W. Edwards Deming, the American credited with the quality and productivity revolution in post war Japan, was regularly quoted as saying that it was good management to reduce the variation in any quality characteristic, whether this characteristic be in a state of control or not, and even if few or no defectives or out-of-specification (OOS) results are occurring (1). To many westerners, this seemed illogical. Why reduce variation if specifications are already being met? However, a short stroll through the quality and management literature of the past 80 years shows a common thread. Deming was not the only one to suggest that reducing variation was beneficial, even though specifications were being met. Dr. John Little, after whom Little's Law is named, proved mathematically the links between throughput volume, work-in-progress, and cycle time. He further demonstrated that conquering variation in process flow reduced cycle time. In turn, work-in-progress must then fall,

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throughput volume must rise, or some combination of both must occur. According to Little, reducing variation pays off because it provides an opportunity to increase throughput from the same mix of people and machines (2).

At the core of Six Sigma is an understanding of variation, rather than ideas such as DMAIC (define, measure, analyze, improve, control), the structured improvement approach, or the "belts" system. The central idea behind Six Sigma is that the variability of any important event in the process ought to be no more than half that allowed by the specifications. Stated another way, the nearest specification must be six standard deviations away from the process mean. Some time before the Six Sigma concept was developed in Motorola, Dr Genichi Taguchi, winner of the 1960 Deming Prize for Individuals, created a concept we call the Taguchi Loss Function (3). The Taguchi Loss Function again demonstrates that Deming was correct to claim that we ought to continue to reduce variation even though all specifications were being met.

Many managers and scientists found a requirement for such demanding precision difficult to understand. Some found it impossible to believe. The mind set in most companies is such that if the characteristic meets specification or standard, no further improvement is necessary; nor is it warranted. In most cases, any result outside of a specification or standard is treated as an assignable or special cause of variation. This, of course, is a mistake.

IN THE BEGINNING...

At the time of the industrial revolution when the first attempts to make interchangeable parts were underway, one breakthrough was the invention of go and no-go gauges. For instance, testing the diameter of an axle might be done with two rings of steel, one slightly smaller than the other. The larger go gauge must fit over the axle, the smaller no-go gauge must not. This breakthrough made possible the manufacture of components in locations far removed from the point of assembly and is a cornerstone of mass production. Soon afterwards, the concept of specifications (or tolerances) was developed. It was at this point that the first definition of quality developed:

All characteristics must remain within the go, no-go specifications.

This is the well-known "goal post" approach to quality. Providing all outcomes stayed between the goal posts (the specifications), all was well. Often pharmaceutical organizations believe that if a result meets specification, not only does nothing more need to be said about it, but also nothing more should be said about it. Ignoring the variation in a key characteristic because it meets specification can be a terrible but all-too-common mistake. Despite 80 years of progress, many scientists and managers find themselves trapped in the old "goal post" mentality.

After Shewhart contributed the notions of statistical stability and invented the control chart, the second definition of quality was developed:

All characteristics must remain stable and within the go, no-go specifications.

This was a significant advance, but both Deming's approach and the Six Sigma methodology are somewhat at odds with this definition, because both are more exacting. With his loss function concept, Taguchi was able to demonstrate that reducing variation below specifications was the best economic alternative. So was the creator of the core Six Sigma concepts, Bill Smith.

In the 1920s, Dr. Shewhart noted that the control of quality was essentially an economic problem. The title of his 1931 book, *Economic Control of Quality of Manufactured Product*, makes this clear. Dr. Genichi Taguchi followed and enhanced this line of thinking. He defined the cost of poor quality as the total loss incurred by society due to variation and poor quality. Taguchi was passionate about quality to the point where he claimed that the manufacturer of poor quality (with particular reference to rework and rejects) was far worse than a thief. When a thief steals $100 from a neighbor, he has gained and his neighbor has lost, but the net economic impact on society is nil. Regardless of who holds it, the $100 will still be invested or spent on goods and services. However, if a manufacturer throws away $100 in rejects and rework, the cost of wasted resources can never be recovered by either the company or by society. In addition, a process with high levels of variation in process flow will have lower throughput that would be the case if the variation in process flow was lower. The difference in throughput translates directly into unit costs. Higher variation in process flow costs money. This loss is permanent.

A useful way to introduce the Taguchi Loss Function is to begin with the supposition that design engineers, chemists, and biologists intensely dislike variation. They prefer perfect precision and have an optimum value in mind. However, these scientists understand that perfection cannot be achieved by mere mortals

Figure 1: The original industrial view of variation.

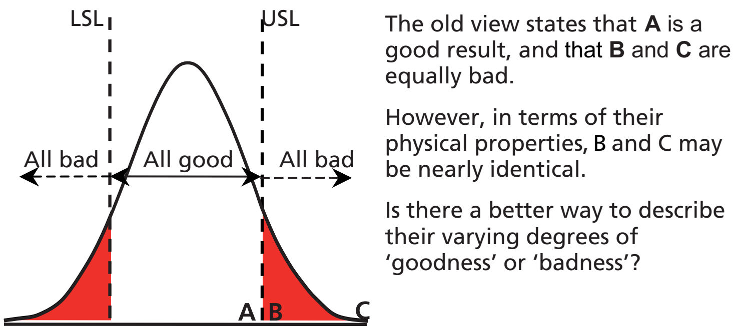
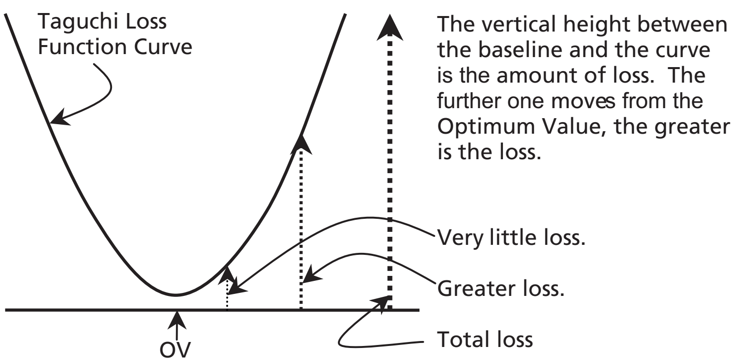


Figure 2: The Taguchi Loss Function Curve.



and the machines they make, and so they create specifications or tolerances to delineate the boundaries of permissible variation. What the designers are telling us when tolerances are set is not how much variation is OK; the designers are telling us what they don't really want but are willing to tolerate. Regardless of this tolerance, there is always an optimum value (OV). For the sake of simplicity and clarity, those characteristics where bigger is always better (i.e., horsepower of a rocket engine) and smaller is always better (i.e., weight of a rocket) will be ignored.

THE OLD GOALPOSTS APPROACH

Imagine we create product A, whose measured characteristic falls only just inside the upper specification, as seen in Figure 1. Now, imagine we measure the next product, product B, which is found to be only just outside the same upper specification. In terms of their physical properties and their performance in the customer's hands, these two products are likely to be, for all practical intents, identical. In most cases, the sampling and analytical error involved will be greater than the observed difference between these two outcomes. Yet the methodology used in most companies will generally state that product B is outside specifications and, therefore, warrants investigation. Now, let us add one more product to the chart, product C, which is well outside the specification. The old go, no-go definition of quality states that all products out of specification are equally bad. In this case, such an approach would lump together outcomes B and C, despite their being very different.

TAGUCHI'S LOSS FUNCTION CONCEPT

As noted in Figure 2, Taguchi envisioned a loss function curve, usually (but not always) a parabola (the simplest form of a loss function curve), which described the increase in loss on a continuum as outcomes moved away from the optimum value, rather than as a go, no-go situation. The vertical height from the baseline (at the baseline there is no loss) to the loss function curve described how the amount of loss increased as results move further away from the optimum value, until eventually, complete loss occurs (2).

For those with a basic grasp of the concept of variation, Taguchi's approach made intuitive sense. Suppose one is using the go, no-go approach and the specifications for pH for a certain process parameter are 7.5 to 8.5. A reading of 8.45 will be accepted as in specification, but one of 8.55 will be out of specification, even though there is no discernable change in the product or in its performance (the sampling and analytical error may well be greater than the difference between these data). The Taguchi approach sees these data not solely on their position relative to specifications, but on a continuum. The target or optimum value is 8.0. Our loss increases continually the further one departs from this optimum. Note how at the OV the loss is minimized, but is not zero. Absolute limits are still necessary for release and compliance reasons, but if loss is seen on a continuum, it opens new perspectives and opportunities to understand and improve the process and product hitherto unnoticed.

The Taguchi Loss Function curve need not be symmetrical, although it often is. Figure 3 shows an example where the curve is asymmetrical. The loss increases faster to the right of the OV than it does to the left. The determination of the shape of the curve is technical in nature, rather than purely statistical.

The Deming, Taguchi, and Six Sigma approaches demand abandonment of reliance on the go, no-go specifications. It is still necessary to set absolute limits, but such limits should not be the sole basis for assessing the variation at any stage of the process. In addition, go, no-go specifications should never be used as the basis for process control.

The Taguchi loss function takes several forms. One quadratic form of calculation is shown in equations 1 and 2 (2).

For any given product:

$$Loss_x = k(x\text{-}OV)^2 \qquad \text{[Equation 1]}$$

Where:

x = value of quality characteristic
OV = Optimum Value
k = A proportionality constant (a function of the failure cost structure)

For any given population of products:

$$Loss_x = k(\sigma_x^{\,2} + (Ave_x - OV)^2) \qquad \text{[Equation 2]}$$

A simplified version of the k factor exists that makes certain assumptions, including that the specifications have been properly set and that any cost of field failure is ignored, is:

k= Cost of failing specification/ (Average - specification limit)^2

UNDERSTANDING LOSSES DUE TO VARIATION

The next step is to overlay the loss function curve onto the relevant distribution. The area-of-intersect of these two curves represents the amount of loss, which can be calculated from equation 2. Some loss function curves are low and flat, allowing considerable movement away from the optimum value before significant loss is incurred. Others are tall and steep, where even a small deviation from the optimum value can incur considerable loss, as noted in Figure 4. The steepness of this curve is driven by the magnitude of the k factor.

In most cases, working on the earlier steps in the process with particular regard to inputs and set-up of the process, provides significant leverage because the loss function curves are nearly always steeper there. The importance of working upstream in the process cannot be overstated. Too often a plant's most talented people can be found working in the middle or latter stages of the process, after variation in the earlier events has already destabilized the process and shrouded cause and effect relationships. Other statistical methods such as DOE will help to isolate

Figure 3: An asymmetrical Taguchi Loss Function Curve.

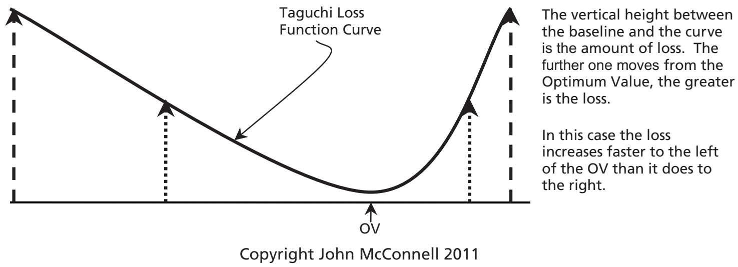


Figure 4: Finding the area of loss.

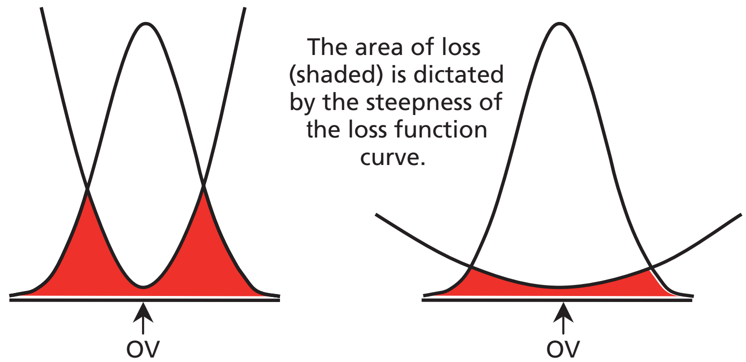
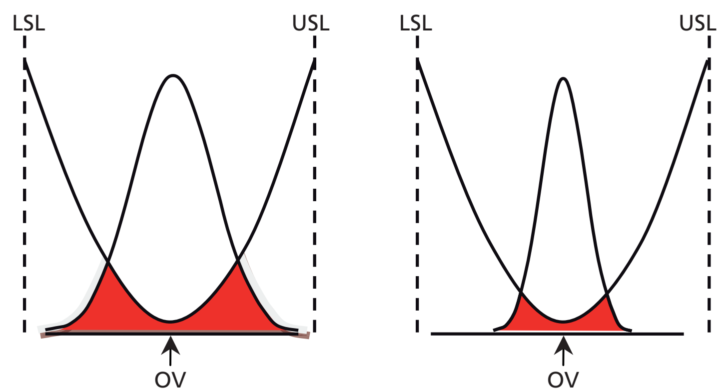


Figure 5: Reducing variation reduces the area of loss.



those variables that have the greatest impact on quality and costs. A cornucopia of advanced statistical techniques exists, but it is beyond the scope of this article to cover them.

Taguchi's approach demonstrates two important ideas. Firstly, it is in concert with Dr. Deming's statement that some loss will be incurred even if all data meet specifications and that businesses should always be attempting to reduce variation. Note how in Figure 5, the shaded area between the distribution and the loss function curve demonstrates that loss exists even though all outcomes meet specifications. Secondly, it demonstrates that as variation is reduced, so too is the amount of loss incurred, as noted in Figure 5. It only remains to isolate those variables that provide greatest leverage.

Taguchi's concepts led to the development of the most complete definition of quality, as follows:

All characteristics should have minimum, stable variation around an optimum value.

Dr. Taguchi's concept is one way to illustrate how variation impacts on processes and costs. Six Sigma is another, as is Deming's approach and Little's Law. What is clear is that several different approaches to the subject arrive at strikingly similar conclusions, as follows:

- Variation costs money, even when everything appears to meet specifications
- Reducing variation always reduces costs
- Not all variables have an equal impact on costs. Some are very significant; others are not.

A critical part of the Taguchi approach is to discover which variables harbor the greatest potential to reduce costs.

Little's Law explains how reduced variation in the flow of material through a process can lead to increased throughput volume, especially when the earliest events in the process are addressed (4). To even the casual observer, it soon becomes apparent that variability impacts on both quality and quantity.

Although the loss function curve can be calculated for any process variable, it is not necessary to do this in order to understand the central ideas and to create progress. Few people using a Taguchi-type mindset actually conduct the calculations. However, it is the concept that is critical, not the calculations. If we understand that variation always adds to costs, and if we are focused on reducing variation everywhere but with particular emphasis on those variables known to have the greatest impact (steepest loss function curves), advancement can be made. If it is grasped that the early steps in the process generally have steeper loss function curves for both quality and quantity, work to reduce variation can commence there immediately. The flow-on benefits throughout the process can be dramatic.

CONCLUSIONS

Variation is not free; not even when all outcomes appear to meet specifications. Variation consumes resources, money, and time. Where a significant proportion of technical personnel's time is expended investigating OOS results, time is wasted that could have been spent improving the process and the product. Reducing variation by achieving a stable state and then minimising variability is particularly crucial in highly regulated industries such as the pharmaceutical and aeronautical industries.

Close enough is not good enough; nor is being satisfied with meeting specifications.

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ARTICLE ACRONYM LISTING

DMAIC	Define, Measure, Analyze, Improve, Control
OOS	Out of Specification
OV	Optimum Value